Asset-Liability Management 2017

Project

Introduction

The project is designed to be solved in groups of 2-3 students. The technical work is the most important. It should result in a report of not more than 15 pages. A neat presentation counts in your favour. Please place important tables and graphs into your report, so as to make it accessible to a reader who does not want to dive deep into Excel workings.

You may write your report in English or Portuguese.

You may contact me with questions at the address walther.neuhaus@zabler-neuhaus.no.

The deadline for delivery of the report and underlying workings is Monday, 30 October 2017.

Have fun!

1. Interest rate analysis

Below are the risk free zero-coupon spot rates for Euro at 31.12.16, annual compounding, according to EIOPA.

Time t	Spot rate
1	-0.302%
2	-0.261%
3	-0.208%
4	-0.123%
5	-0.024%
6	0.092%
7	0.215%
8	0.341%
9	0.461%
10	0.571%
11	0.671%
12	0.760%
13	0.841%
14	0.908%
15	0.958%

1.1 Determine the term structure with continuous compounding.
1.2 Determine forward rates that are consistent with the zero rates.
1.3 Produce a graph showing the zero rates and forward rates.

2. Liability valuation

For a certain insurance portfolio, your actuary reports the following outstanding claims and predicted payments:

Time t	Liability cash flow
1	6 527 416
2	4 801 976
3	3 772 174
4	2 841 440
5	2 224 827
6	1 724 919
7	1 220 471
8	823 638
9	595 814
10	388 856
11	274 339
12	167 101
13	96 047
14	48 275
15	20 081
Total	25 527 374

For the sake of simplicity, you may assume that payments fall due at the end of the year.

2.1 Calculate the discounted value of the liability cash flow, using the EIOPA yield curve.
2.2 Calculate the duration and the convexity of the discounted liability cash flow.
2.3 Using duration and convexity, estimate the change in the discounted value of the liabilities that would occur if zero rates were to change by 0.5% up or down, across the length of the yield curve.

3. Matching and immunisation

Assume that bonds of every maturity are available and that they all have a coupon rate of 2%.

3.1 Find the portfolio of bonds that matches the expected liability cash flow.
3.2 Recommend a portfolio that combines three bonds in such a way as to match the present value, duration and convexity of the liabilities. Negative holdings are not allowed.
3.3 Create three "buckets" of bonds: Short (maturity 1-5 years), Medium (maturity 6-10 years) and Long (maturity 11-15 years). Each bucket contains one piece of each bond in the specified maturity range. Determine the mix of the three buckets that matches the present value, duration and convexity of the liability cash flow.

4. Cost of capital

In 2.1 you calculated the discounted value of the liability cash flow, using the EIOPA yield curve. That is called the "best estimate of claim provisions" in Solvency II parlance.

Assume that the capital required to hold the liabilities is 10% of the remaining best estimate of claim provisions, now and at the end of every future year. The cost of capital is set at 6%.

4.1 Predict the best estimate of claim provisions now, and at the end of every future year.
4.2 Predict the capital required to hold the liabilities now, and at the end of every future year.
4.3 Calculate the cost of capital for every future year.
4.4 Calculate the overall cost of capital, being the present value of all future cost of capital.

5. Mean-Variance analysis

Your investment mandate is restricted to three asset classes: Equity, Bonds and Money market.

Use the following set of assumptions for the yearly asset returns:

Asset	Asset class	E(return)	SD(return)	Correlations	1	2	3
1	Equity	6.00 %	8.00 %	1	100.0 %	16.0 %	15.8 %
2	Bonds	1.50 %	0.50 %	2	16.0 %	100.0 %	67.7 %
3	Money market	0.50 %	0.20 %	3	15.8 %	67.7 %	100.0 %

Assume the risk free rate is 0.1%.

5.1 Use the mean-variance structure to determine, with a one-year time horizon:

1. $\mathbf{w}_{\min}$: The minimum asset variance portfolio,
2. $\mathbf{w}_{\text{ref}}$: The reference portfolio of risky assets,
3. $\mathbf{w}_{\tan|\mu_0=0.1\%}$: The tangency portfolio when the risk-free return is 0.1%.

5.2 Now assume that liability growth R_L has the following stochastic behaviour:

$$\begin{aligned}
\mathrm{E}(R_L) &= 1.5\% \\
\mathrm{SD}(R_L) &= 0.5\% \\
\mathrm{Corr}(R_L, R_{Equity}) &= 10\% \\
\mathrm{Corr}(R_L, R_{Bonds}) &= 40\% \\
\mathrm{Corr}(R_L, R_{Money\ market}) &= 25\%
\end{aligned}$$

Assume that the funding ratio is $F = 110\%$ and that the required asset return is $r = 1.5\%$.

Calculate $\gamma = \mathrm{Cov}(\mathbf{R}_{Asset\ classes}, R_L)$ and then

4. $\mathbf{w}_\gamma$: The liability hedge portfolio.
5. $\mathbf{w}_{\min}(F, \gamma)$: The minimum surplus variance portfolio,
6. $\mathbf{w}_r(F, \gamma)$: The surplus-efficient portfolio of risky assets,
7. $\mathbf{w}_{r|\mu_0=0.1\%}(F, \gamma)$: The surplus-efficient portfolio when risk-free return is 0.1%.

5.4 Discuss the feasibility of the efficient portfolios $\mathbf{w}_r(F, \gamma)$ and $\mathbf{w}_{r|\mu_0=0.1\%}(F, \gamma)$. Can they be realised in practice?

5.5 Do 1000 simulations of the one-year asset returns as well as the liability growth, using a multivariate normal distribution that has the specified means and covariance structure. At the end of the simulation, find the means and covariance matrix of the simulated data and compare them to the assumed means and variances, as a check that your simulation is working correctly (they should be close).